In the assessment of the interacting spatial automata formalism presented here, I conclude that the focus on interactions and rigorous construction methods provides users of the formalism with a framework under which to model complex spatial systems as discrete event systems. Using the formalism, interactions between components are modelled explicitly and independently from delays. In addition, hierarchical models may be built using a reversible abstraction operator, which has been implemented for spatial abstractions in a wildfire model. Together, these two feature of the formalism demonstrate the ability to model multiresolution spatial heterogeneity in an ecological setting; by mimicking the multiscale patchiness of fuel distributions in natural environments.

The interacting spatial automata formalism builds models of spatial event-driven systems by composing individual cells to form a set of interacting processes. The methodological framework developed through experiments in Chaps. 6 and 7 is a
demonstration of how the formalism is used to implement a model of wildfire dynamics. In Chap. 6, the presented method shows how it is used to construct a model of wildfire in which the influences of heterogeneous wind and slope on fire behaviour are captured. The results highlight the limitations of existing models of wildfire behaviour. In Chap. 7, a novel structure is introduced where the cells in the system are built as a hierarchy and use an irregular discretisation. In addition, the experiments demonstrate the difference between existing propagation mechanisms and the neighbourhood influence propagation mechanism, showing that the neighbourhood influence mechanism may be used to model wildfire behaviour.

The Circal process algebra is used as a specification language in the interacting spatial automata formalism and it has four significant advantages. Circal builds models of concurrent systems and the methodology presented in Chap. 7 demonstrates how the Circal specification may be translated to a sequential event-scheduling discrete event system. A model constructed as a set of connected cells (specified by Circal processes) is a general approach to modelling the spatial properties of a system in a discrete way. Using Circal descriptions of connections and labelled events, interacting between components in a model are captured explicitly. The abstraction mechanism for building a hierarchical connective structure in a model uses the Circal abstraction operator.

The significant contribution to computer science presented in this thesis is the alternative approach to building constructive models of wildfire dynamics, using the interacting spatial automata formalism. The approach to modelling spatial event-driven systems is novel because it uses the Circal formalism, which allows it to model interactions between discrete components (cells) explicitly. A novel structure that is both hierarchical and irregular is implemented to describe a natural fuel distribution. The irregularity of the structure allows for isotropic propagation and the preliminary findings of the method of abstraction suggest that the hierarchical structure is a viable alternative approach to modelling systems that exhibit
multiscale heterogeneity.  1

The significant contributions presented in this thesis are in line with the aims of producing more physically realistic models of wildfire propagation through natural landscapes. The purpose for building models of wildfire dynamics is to reduce social and economic losses including the loss of life and property. A wildfire simulation may be used to provide real-time analysis of a wildfire event or assist in the risk analysis of specific natural landscapes and the interface between the natural environment and agricultural/urban landscapes. In the research presented in this thesis, the constructive and interaction-centric approach highlights the need for more physically realistic specifications of wildfire models.

8.1 Advantages of the interacting spatial automata formalism

The interacting spatial automata formalism represents an alternative approach to modelling spatial systems because it uses the spatial discretisation of a generalised cellular automaton but specifies the structure and behaviour of the model using the Circal process algebra. The formalism allows for the explicit modelling of interactions, where labelled events are synchronised to describe the behaviour of a model during a simulation. Spatial systems are modelled by a generalised spatial discretisation; cells represent a specific area of a two-dimensional landscape. Concurrency is managed by the synchronisation of events, cells change state when an event is shared with its neighbour. A rigorous method of abstraction, afforded by the Circal specification, allows for the construction of hierarchical models.

1The preliminary findings suggest that the hierarchical structure is a viable alternative approach to modelling systems that exhibit multiscale heterogeneity, but it was demonstrated only for the specific application domain and in specific experiments within the thesis.
8.1.1 Explicit modelling of interactions

Circal processes model interactions between processes explicitly via labelled events. In the interacting spatial automata formalism, the explicit modelling of events also defines the structural connections between cells and the connectivity. The interacting spatial automata formalism uses an alternative method for dealing with interactions to that of a cellular automaton or a Cell-DEVS model.

In Chap. 5, the interacting spatial automata formalism is developed; the structure in Sec. 5.2.1 and the behaviour in Sec. 5.2.2. The structure of an interacting spatial automaton is a set of connected cells, where connections are determined by the set of cells that share events. The behaviour of a cell may be described by a finite state machine, where transitions are given as the labelled events that correspond to each connection with the cell’s neighbourhood. As a simulation progresses, the transitions internal to a cell (and thus specifying the behaviour) are caused as a result of synchronisation of events between cells and this subsequently creates an ordering of events for the entire cellular space. In this way, the behaviour of the entire model is dependent on the interaction between cells created by the labelled events.

Cells in the wildfire model described in Chap. 6 have unburnt, burning and burnt states as well as intermediate states used for the purpose of a timing mechanism. The events of the model include an ignition event and a clock tick event. In Chap. 7, the timing mechanism is separated from the finite state machine specification of each cell and each cell is connected to its neighbours via an ignition event. In this model, an ignition is an event that is shared between an unburnt cell and the set of burning cells in its neighbourhood.

This approach offers an alternative method of specifying the structure and behaviour of a set of interacting components. In cellular automata [66, 67, 36], the interaction between cells is inherent in the transition function where neighbourhood state is used to define the new state of a cell. In Cell-DEVS models [111, 112], delay values
are explicitly attached to each event which is described by a causal relationship between two cells; one cell causes an event to occur in another cell. The interacting spatial automata formalism is different to a cellular automaton because labelled events are associated with connections between cells. The interacting spatial automata formalism is different to a Cell-DEVS model because connections between cells may exist between a set of cells rather than just a pair, and the delays may be implemented in different ways (as in the timing mechanism described above).

8.1.2 Generalised spatial discretisation

The interacting spatial automata formalism extends the capability of discrete spatial modelling to allow for arbitrary representation and connectivity. Typically, models of spatial systems use regular and square grids of static connectivity as used in Chap. 6. The limitations of such methods are practical (deterministic implementations on regular grids do not model isotropy) and theoretical (the use of regular and square lattices is based on human reliance on ease of implementation rather than the modelling of nature). Since models specified using Circal model explicit interactions between processes that may vary in behaviour, the representation of a cell (such as the area and shape) and its connection to its neighbours may vary. In Chap. 7, an irregular spatial discretisation and connectivity is specified using the formalism and is demonstrated to model isotropy, which is an important characteristic of fire behaviour in continuous environments.

The interacting spatial automata formalism models a more general spatial discretisation than cellular automata because it may specify arbitrary connectivity between cells. Connections between cells in a cellular space are associated with labelled events and the neighbourhood of a cell is specified by the set of cells to which it is connected. A cellular space may specify an arbitrary connectivity where each cell

\footnote{Another advantage of an arbitrary connectivity is the ability to model longer range interactions such as a simple model of spotting, where ignition events may be shared between cells at a geographical distance. This is not implemented in any of the models described here.}
is associated with a different number or configuration of neighbours.

In Chap. 6, cells adopt the typical approach using regular square cells and a homogeneous connectivity, where each cell is connected to its sixteen nearest neighbours. This is a typical approach and has been used in several existing implementations of wildfire models (for example, by Lopes et al. [18] and Hargrove et al. [13]). The advantage of using a regular and square grid as a cellular space structure is an ease of implementation; computers and programming methods are suited to building models as discrete two-dimensional arrays.

In Chap. 7, cells represent a polygonal patch of fuel; each patch may have a different shape and different number of neighbours with different distances associated with each connection. This is a generalisation of the typical cell-based approach to modelling spatial event-driven systems. Using the generalised spatial discretisation, an irregular cellular space is defined through the use of Delaunay triangulation and Voronoi decomposition [127]. This cellular space structure is a closer representation of the patchiness of natural fuel distributions than the regular square lattice of the Chap. 6 implementation.

Isotropy is a property of natural wildfires that is not captured by cell-based models that are deterministic. Hargrove et al. [13], for example, introduces a probabilistic update for diagonal cells in a regular grid to produce isotropy. In another example, Sullivan et al. [14] introduce a random wind variation to average out the effect of polygonisation in a model of wind influence on wildfire behaviour. In Chap. 7 the non-determinism of the approach is implicit in the structure, due to the random distribution of nodes over the cellular space. The simulations presented in the chapter demonstrate that isotropy is modelled by the irregular structure afforded by the generalised spatial discretisation of the formalism.
8.1.3 Managing concurrency

Concurrency management in the interacting spatial automata formalism is achieved through the synchronisation of events, afforded by the Circal specification of cell behaviour and connectivity. The approach is related to process interaction techniques of discrete event systems methods [72, 22]. A cellular space is constructed by specifying individual cells and building connections between them via labelled events. The formalism manages the ordering of events by making local compositions; it finds events that are current within neighbourhoods and updates the state of the involved cells.

In Chap. 6, a clock process is specified that separates events by regular clock ticks. The clock process is connected to every cell in the cellular space and has the effect of guarding events. In Chap. 7, a global queue is maintained separately to the cellular space, where active events are arranged in the queue depending on the state of the neighbourhood from which they originate. The global queue manages the ordering of events and events removed from the head of the queue are used as the transitions between states in cells. The global queue achieves the management of events by assessing the set of active cells and their neighbourhood in order to schedule when a synchronisation of events should occur.

In process interaction methods in discrete event systems, individual components within a model use an internal mechanism to determine the timing of their state transitions. In event-scheduling and activity scanning methods in discrete event systems, an external mechanism predetermines or scans (respectively) the set of components in the model to determine an ordering for events. In asynchronous cellular automata [69], timing is specified before simulation by describing an ordered set of updates for individual cells. The interacting spatial automaton specified in Chap. 6 implements a method that is similar to asynchronous cellular automata and also process interaction techniques. The ordering is not determined before the simulation (as in an asynchronous cellular automaton as defined in Sec. 5.1.3), but
the clock process views the state of the cellular space in order to determine the ordering of events in the next step. In Chap. 7, the clock process is replaced with a global queue that stores real valued delays associated with the set of possible future events within a non-quiescent neighbourhood. This model implements a continuous time discrete event method [70].

The methods used in implementing the ordering of events adapts the Circal method for simulation as opposed to verification, which is the traditional purpose of Circal. During a simulation, a cellular space may be segmented as a set of active cells and a set of quiescent cells. The neighbourhood around a set of active cells defines the region of a cellular space which may be active during the next step of a simulation. In verification methods, a Circal model discovers all the possible orderings of events in order to say something useful about the properties of the system. In simulation methods, only one ordering of events is considered and this means that a simulation can progress by choosing an ordering for events at each time step during a simulation. This is why only the neighbourhood of active cells needs to be considered at each point in a simulation. As a result, the interacting spatial automata formalism is capable of managing the concurrency of a set of interacting components in an alternative way to that of the Circal formalism.

8.1.4 Abstraction and hierarchy

The interacting spatial automata constructs hierarchical models via abstraction and concretisation methods. The Circal process algebra uses composition and abstraction operators to build hierarchical models; higher level processes are constructed by composing lower level processes. This is achieved by internalising connections as described in Sec. 7.1.1 The abstraction mechanism does not remove the need for modelling choices. In Chap. 7, one method for building a hierarchical model of a spatial system is described. In this approach, connections are removed and added to produce higher and lower resolution representations of landscape segments. It is concluded that the chosen method is limited by the manner in which cells change
the area they represent during abstraction; the method is not scale-invariant.

A method for building hierarchical models is introduced in Sec. 5.2.1 and implemented in Chap. 7. The Circal formalism’s abstraction operator allows a modeller to hide events (and therefore connections) in a cellular space. Using the abstraction mechanism an entire cell may be hidden from the model when it is no longer connected to the cellular space (all of the events it shared with its neighbours are abstracted). 3 In a continuous landscape, cells are contiguous and when a cell is abstracted from the cellular space neighbouring cells are extended to cover a greater area. This is the reason abstractions are performed over internally homogeneous neighbourhoods; the properties of different fuels need not be aggregated.

The change in connectivity caused by abstraction has an adverse effect on the isotropy of propagation for some cellular space configurations. Despite the conservation of internal homogeneity in abstraction, the connectivity of the landscape changes during the abstraction process. In Chap. 7, the Delaunay triangulation [127] of nodes was chosen to build the connective network in the cellular space and it defines the connections and neighbourhoods. When abstraction occurs, the Delaunay triangulation of the cell space with the node removed specifies the new connections. The Voronoi decomposition of nodes described the extent (area) of cells around the nodes. When an abstraction is made, the Voronoi decomposition of nodes with the node removed specifies the new areas of the cells in the neighbourhood. The result of an abstraction is a change in connectivity and a change in area for the neighbourhood of the disconnected cell. The change in connectivity may have a specific orientation that, when aggregated with other abstractions, causes an overall bias in orientation for the connections (see Fig. 48 for example). As a result of this bias in orientation, the isotropy of propagation is lost because the distances are more inaccurate (distances increase) in some directions.

---

3The cell still exists in the cellular space, but it is no longer connected or visible. If the information about an event is ‘remembered’ by the model, the abstraction process may be reversed by replacing the lost events.
8.2 Properties of spatial event-driven systems

In Sec. 1.1, several properties of spatial event-driven systems were highlighted as significant to modelling and simulation. These included heterogeneity, locality and model-specific spatial distribution. Heterogeneity is important to modelling spatial event-driven systems because the subtle irregularities of a system may be responsible for the global behaviour due to feedback mechanisms that exist in the system’s behaviour [109]. Locality is important to models of spatial event-driven systems because it reflects the nature of a physical system where activity is propagated through a space via local interactions between components. Model-specific spatial distributions are important to spatial models because the traditional method of using regular square cells may not always be the most appropriate method of spatial discretisation. Rather than trying to fix the inaccuracies after creating a regular cellular space, a model-specific spatial distribution attempts to capture the underlying structure of a system using irregular cells and hierarchy.

Since the interacting spatial automata formalism is capable of modelling explicit interactions between cells, it is also capable of modelling heterogeneity. Each cell is specified with an individual set of states and transitions (transitions are events shared with its neighbours). A simple example of heterogeneity is given in Sec. 5.1.1. In Chap. 6, heterogeneous cells are specified by different values for wind and slope, each cell specifies a (possibly) different order and set of delays for which it ignites it neighbours. In Chap. 7, cells are heterogeneous because they each represent a different area and may represent a different fuel. Depending also on the distances between the cells, different areas and fuel types may increase or decrease delays to ignition in neighbouring cells.

The difference between a line fire and a point fire is captured by the neighbourhood influence propagation mechanism introduced in Chap. 7. The neighbourhood influence propagation mechanism is an alternative mechanism that uses the ability of Circal to specify connections between sets of cells rather than pairs. In Chap. 7,
the ability to model locality in this way is used to describe a physically realistic parameterisation of influences; the effect of a neighbourhood of burning cells in a neighbourhood is taken into account for the time to ignition of a cell (rather than just using information about the closest burning cell, for example).

Since the interacting spatial automata formalism can model a general spatial discretisation, it may be used to model the spatial properties of a spatial system in alternative ways that may also be more physically realistic. Where traditional methods use square and regular grids to describe the properties of a spatial system, the interacting spatial automata formalism may model spatial systems as irregular and hierarchical discretisations of space. In Chap. 7, the irregularity of the cellular space is achieved by distributing nodes over a landscape at random (but constraining the placement using a minimum distance between points as in Schönfisch [68]). The hierarchy of the cellular space is achieved by applying the abstraction mechanism to disconnect cells and redefine their neighbours to represent the area of the disconnected cell.

8.3 A summary of contributions

The contributions detailed in this work includes a novel structure for cell-based modelling of wildfire dynamics that extends generalised cellular automata to provide a model of natural patchiness of vegetation and a new modelling formalism that uses a set of rigorous operators to implement more general models with a focus on interactions. In addition, the hierarchical and irregular structure of the wildfire model exhibits isotropic propagation in homogeneous conditions. The method of model construction by the synchronisation of events provides a user with rigorous mechanisms for specifying the behaviour of individual cells and composing them such that the ordering of events is preserved globally; it extends asynchronous cellular automata methods to provide a cellular space with an update mechanism that is defined internally rather than specified externally as a static order.
The interacting spatial automata formalism is significant because it offers a method of specifying spatial, event-driven systems in a rigorous manner, with a focus on interactions. The formalism provides an alternative to generalised cellular automata and Cell-DEVS and is appropriate to modelling wildfire systems.

The irregular structure is a novel approach in models of wildfire propagation, particularly where propagation is built for heterogeneous environments. It is adapted from methods in generalised cellular automata theory, where it is used to model isotropy in homogeneous and simplified models of propagation.

The hierarchical structure (used to implement multiple resolutions of fuel distributions) is significant to both generalised cellular automata and wildfire modelling. Whilst multiple resolutions have been used in cellular automata previously, a spatial hierarchy and a rigorous mechanism for abstracting between levels has not previously been implemented.