CELL-DEVS MODELING AND SIMULATION OF ARTIFICIAL HYDRAULIC FRACTURING OF ROCKS IN BOREHOLES (WIP)

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ABSTRACT

Hydraulic Fracturing is a technique used in the extraction and exploration of oil and gas. During hydraulic fracturing, a high-pressure fluid is injected into a wellbore; this fluid flows through some artificially made holes and into some rock fragments. As a result of the high pressure of the fluid, the rock fragments crack or fracture. These fractures form patterns that grow in a complicated manner. In practice, hydraulic fracturing is widely used; therefore, people and companies have a great interest in understanding the fracturing process. They aim to predict where the new fractures will be located and what the length, width and shape of the fractures will be. A model of hydraulic fracturing using the Cell-DEVS paradigm is described and implemented, discussing a discrete-event specification of such application.

Keywords: Hydraulic Fracturing, Cell-DEVS, State variables

1 INTRODUCTION

Hydraulic fracturing is a process used to enhance the production of oil and gas. This process is widely adopted in the oil and gas industry for extracting oil and gas resource, trapped in underground reservoirs called wellbores. The idea is to extract oil and gas from underground reservoirs using a frac-fluid (consisting of water, sand and some chemicals) pumped at a high pressure into a selected section of the wellbore. Hydraulic fracturing produces cracks by pumping the fluid at a relatively high speed and pressure into existing or artificially made rock crevices. The size of these fractures can range from several meters to hundreds of meters, and their cost is often an important part of the overall cost of oil exploration exercises. The pressure of such fluid creates fractures, and these extending into the rock medium. If the rock contains oil or gas, the high-pressure fluid will reach the mineral, which can then be extracted (Yew and Weng 2014).

The frac-fluid is a high-pressure liquid which causes hydraulic fracturing. Hydraulic fracturing in oil and gas reservoirs often leads to a development of complex fracture networks. It is an important venture for the oil and gas industry to understand the nature and extent of the complexity of ensuing fractures, to optimize the strategies for designing and completing exploration exercises, having an idea of possible outcomes. Fractures come about as a result of the action of existing high-pressure fluids in underground reservoirs called wellbores, which exert a force proportional to their inherent pressure on surrounding rock bodies. The extent of their complexity is analogous to a nerve network structure of varying length, width and height combinations.

Different earlier research has used finite elements to define hydraulic fracturing models, and here we propose a new method to model and simulate the process of how the frac-fluid cracks the rock(s). We want to be able to represent the fracture pattern that results from hydraulic fracturing using Cell-DEVS, in order to be able to use the different advantages provided by DEVS and Cell-DEVS (Wainer 2009).

The rest of the paper is organized as follows: Section 2 has the related work, Section 3 has the definition and implementation of the hydraulic fracturing model with some results, and finally, Section 4 is the conclusion.

2 RELATED WORK

Many authors have studied hydraulic fracturing process via numerical simulations; a historical background of the development of hydraulic fracturing models is provided by (Adachi et al. 2007). There are two basic numerical constant height models which were used to implement hydraulic fracturing in the past: the Khristianovic-Geertsma-de Klerk (KGD) model which is valid under plane strain and the Perkins-Kern-Nordgren (PKN) model which is valid under plane stress.

For the PKN model, "in addition to assuming a constant fracture height, the other two assumptions are (1) the fracture is at a state of plane strain in the vertical plane and the vertical fracture cross-section is elliptical and (2) the fracture toughness has no effect on the fracture geometry." (Yew 2014). For the KGD model, "in addition to the constant height assumption, two other assumptions are (1) the fracture is at a plane strain condition in the horizontal plane and (2) the fracture tip is a cusp-shaped tip." (Yew 2014). Another numerical model used to study hydraulic fracturing process is the radial or penny-shaped model with constant fluid pressure was solved by (Sneddon 1946).

(Pan et al. 2011) uses a cellular automaton (CA) approach to generate the single fracture structure, which is assumed to be composed of contacts and voids. "Natural fracture properties such as dead voids, islands and tortuous flow path are reflected. CA updating rule to simulate fluid flow in a fracture with contacts is developed. They conclude that fracture flow behavior strongly depends on the effective fluid flow path." (Pan et al. 2011)

The hydraulic fracturing model was implemented with CD++, a tool used to implement DEVS and Cell-DEVS models. Cell-DEVS (Wainer 2009) is a formal modeling and simulation methodology that allows defining discrete-event cellular models with explicit timing delays. Each Cell-DEVS atomic cell holds state variables and a computing function to update the cell's state. This is done by using the present cell state and those of a finite set of nearby cells (called neighborhood). The efficient computation of cell-state variations allows one for developing complex models, and it provides straightforward integration of the models with other modeling formalisms.

CD++ (Wainer 2009) is an open source M&S tool that provides a development environment for implementing Cell-DEVS models using a built-in specification language based on the formal specifications of Cell-DEVS, including the size and dimension of the cell space, borders and the shape of the neighborhood. The cell's local computing function is defined using a set of rules with the form POSTCONDITION ASSIGNMENTS DELAY {PRECONDITION}. When the PRECONDITION is satisfied, the state of the cell will change to the designated POSTCONDITION, whose values will be transmitted to other components after the DELAY. If the precondition is false, the next rule in the list is evaluated until a rule is satisfied or there are no more rules. If model's state variables need to be modified, the ASSIGNMENTS section can be used. CD++ interprets this specification language and executes a simulation of the model. The simulator also allows the use of multiple neighbor ports for each cell in a Cell-DEVS model.

We present two implementations of modelling the hydraulic fracturing process using the Cell-DEVS formalism in this paper. Unlike the complex rules and/or equations used in most approaches, we use simple rules to model the hydraulic fracturing of rocks.

3 HYDRAULIC FRACTURE MODEL

We want to define a model of hydraulic fracturing using Cell-DEVS formalism in order to experiment with a different method. We defined different models that are explained in this section. All of them are designed as a 2-dimensional cross section of an underground hydraulic fracturing event, located immediately next to the wellbore. The scope of the model is the dotted rectangle shown in Figure 1, where the injection point of the fluid is referred to as the "*well spout*".

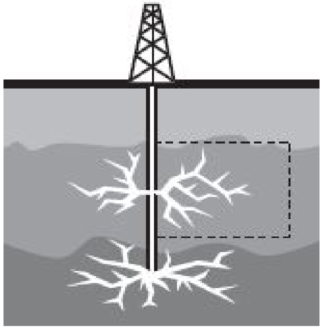

Figure 1: Hydraulic Fracturing process showing fracture patterns

Possibly, the geometric accuracy of hydraulic fractures can be predicted and controlled in locations where the *in-situ* stress field, the direction and the position of one of the wells are known.

A rock cell (a cell which has not been fractured) will have zero pressure and a fractured cell (a cell is fractured when the rock material has broken down and admits hydraulic pressure) will have a pressure greater than zero. Finally, the well spout behaves like a fracture, but it continuously admits hydraulic pressure into the system. The pressure is increased continuously and it is transferred to other places that have been filled with liquid. The state, pressure and density of each rock cell are considered.

The hydraulic fracturing model was implemented with CD++. We discuss two implementations below.

3.1 CD++ Implementation of the Hydraulic Fracturing Model

The Hydraulic fracturing model is defined as a 2D cell space with 2 planes, each plane contains 625 cells. Each cell represents a small fraction of rock, and all the cells together form a rock fragment. Consider each cell to be a small fraction of rock and all the cells together form the rock fragment in the dotted rectangle shown in Figure 1. In the model, each plane contains some information about the cells or the small fractions of rock. Plane 0 contains the pressure of each cell and Plane 1 contains the direction of each cell (this will be explained later). The 2 planes that make up the model are explained further below.

Plane 0: The state value of each cell in this plane represents that cell's hydraulic pressure.

- A value of zero means that the cell is a rock: it is occupied by solid material that is not yet fractured but can be affected by pressure exerted by fractured neighbors.
- A value greater than zero means that the cell is fractured. The pressure value is in Pascals. The rock material has broken, and it admits hydraulic pressure. Rock cells have a threshold at which they can break and become fractured cells, called the *maximum supported pressure*. The formula used to calculate this value will be provided later, and it accounts for depth under the earth's surface, which in the cell space equates to the vertical position of the cell within the space. In this model, the maximum supported pressure of a rock is multiplied by that rock's *resistance factor*. The resistance factor increases with the number of neighboring fractures. This represents the rock being harder as it is compressed by the fractures opening nearby. The direction influences if a rock cell is fractured or not.
- The 'well spout' cell behaves like a fracture, but it continuously introduces hydraulic pressure into the system. Hydraulic pressure tends to dissipate across adjoining fracture cells. In this way, fractures transmit hydraulic pressure from the well spout throughout the cell space. The well spout can be thought of as the cell through which fluid is pumped into the rock fragment.

Each cell in Plane 0 uses a Moore's neighborhood of 9 cells (including itself) in Plane 0 and a Moore neighborhood of 9 cells in Plane 1. A diagram showing the neighborhood of each cell in Plane 0 is given in Figure 2 below.

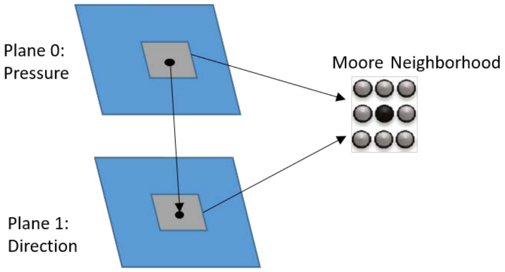

Figure 2: Neighborhood of cells in Plane 0.

Plane 1: Cells in this plane can have 4 states: 0, 1, 2 or 3, representing a direction. The idea is that if a cell in the neighborhood of a rock cell has enough pressure to fracture the cell, the rock cell will only be fractured if the neighborhood cell has the same direction as the rock cell. In other words, if cell B (in the neighborhood of cell A) has enough pressure to fracture cell A, then cell A will only be fractured if the direction of cell A is equal to the direction of cell B. Each cell in Plane 0 has a corresponding direction in Plane 1.

The direction rule is that a rock cell will fracture only if it has the same direction as the neighborhood cell that will cause the rock cell to fracture. This rule is used to ensure that the hydraulic fractures will not be disjointed but connected together to represent the frac-fluid flowing between two fracture cells and the overall fracture.

Once the coupled model definition has been specified, we need to define the rules for each of the cells. We have different rules representing different possible behaviors in the cells, described below.

1. **Rock rule:** A rock cell will become a fracture cell if the neighborhood cell's pressure exceeds the product of its maximum supported pressure and its resistance factor. In addition, the neighborhood cell must have the same direction as the rock cell. The maximum pressure support of the rock cell is:

$$\text{MaxPressureSupport} = p_d = 2\sigma - p_f + T_0$$

Where σ is the smallest principal stress, p_f is the pore fluid pressure, and T_0 is the tensile strength of the rock. Each cell has its own pore fluid pressure and its tensile strength. They are computed as described in Figure 3 below.

simbol	description	units	value
ρ	rock density	$\mathrm{kg\ m^{-3}}$	2200
T_0	rock tensile strength	Pa	$15 \pm 1 \times 10^6$
g	gravity acceleration	$\mathrm{m\ s^{-2}}$	9.8
gP	pore fluid pressure gradient	$\mathrm{Pa\ m^{-1}}$	10500
p_f	pore fluid pressure	Pa	$\mathrm{depth(m)} \times gP$
σ	stress	Pa	$\mathrm{depth(m)} \times g \times \rho$

Figure 3: Definition of variables used to compute MaxPressureSupport.

p_f and σ depend on depth, so they will have different values as depth increases. In the model, we assume that the fracture is taking place 3048 m below the ground. The resistance factor of the rock cell is computed as: $R + s * p$. Where R is the resistance multiplier minimum, s is the resistance multiplier range span, and p is the percentage of fractured non-self neighbors. This formula proceeds as follows:

$$1 + 10 * (8 - (\text{number of cells which are zero or undefined})) / 8.$$

2. **Fracture rule:** If a cell is a fracture cell and the cell state in plane 0 has a value greater than 0, the pressure of that cell is the average pressure of all the neighborhood cells that are fracture cells. In any other case, the cell keeps its current value.

3. **Well spout rule:** The well spout cell follows the fracture rule but is also constantly incrementing its pressure by 10 MPa.

4. **Zone Direction rule:** Each cell in Plane 1 (direction plane) is allocated a random value between 0 and 3 every 100 ms.

When we simulate this version of the model, the well spout cell starts off as a fracture while the rest of the cells are rocks. The extreme left cell space in Figure 4 below shows the initial state of the cell space. In Plane 0, blue cells represent fractured cells and dark brown cells represent rock cells. The well spout cell continuously introduces pressure to the cell space. Eventually, when the pressure of the well spout exceeds that of some of its neighborhood cells and the neighborhood cells have its same direction, these neighborhood cells become fractured. The extreme right cell space in Figure 4 below shows the cells which are fractured next due to the well spout's pressure. Notice that the fractured cells are connected to each other and they already show a fracture pattern. A later stage of the fracturing process is shown in Figure 5. Notice the more complex fracture pattern evolving in Plane 0.

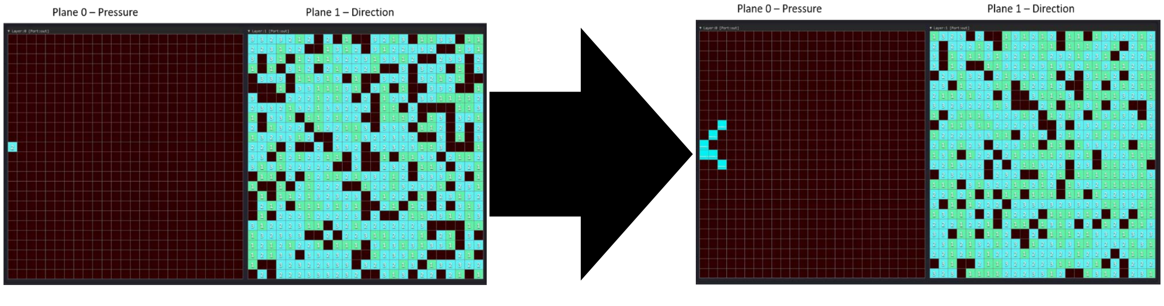

Figure 4: Initial Values of the Cell Space and the Next stage of the fracturing process.

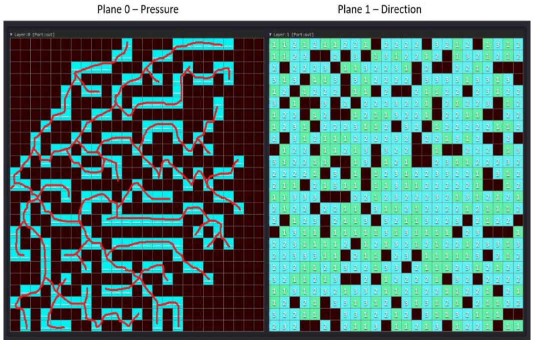

Figure 5: Fracturing process.

In order to properly see the fracture pattern, we traced the blue cells which were connected to each other. The red lines in Figure 5 show the intricate fracture pattern created by the fractured cells which are a by-product of the hydraulic fracturing process which the model attempts to represent.

3.2 Single-plane Hydraulic Fracturing Model

In this section, we present an extended version of the model using multiple state variables and multiple neighbor ports for each cell in a Cell-DEVS model. The CD++ implementation included two extra planes which represented state variable mappings to the cells in the plane-0, "the pressure plane". These planes are compressed into a single plane, each cell having state variables and multiple state ports. The state variables represent the planes in the Cell-DEVS model. In this implementation, we use cell-spaces of 50x50 and 100x100. Each cell uses three neighbor ports to transmit values for pressure, state and direction. These neighbor ports are called are called as "pressure port", "state port" and "direction port" and they are described below.

- **State port**: Holds a value corresponding to the three possible occurrences of a cell. A cell will either exist as a rock, a fractured cell or a well spout. A value of 0 represents that a cell is a rock, 1 represents that a cell is a fractured cell and 2 represents that a cell is a well spout.
- **Pressure port**: Holds the value corresponding to the instantaneous pressure of a cell per time, and makes this available to neighboring cells. 0 indicates that the cell is yet a rock and the pressure for a rock and none of its neighbors have been able to overcome its resistance factor.
- **Direction port**: Holds the value corresponding to the direction of a cell. This value is a random integer output from a random generator function and is made available to a cells neighborhood cells. The range of values of a cell's direction is in the region [0, 8]. Each cell – a rock cell especially, has a direction which is used in conjunction with other parameters (pressure port and state port) to determine if a rock cell will be fractured or not.

The threshold at which rock cells fracture is computed as in Section 3.1. The formula represents the depth below the surface of the earth, which in the cellular space corresponds to the vertical position of the cell in space. But in this case, the maximum pressure supported by a rock is multiplied by the strength factor of that rock whereas the resistance factor increases with the number of neighboring fractures. This represents the rock that becomes stronger because it is compressed by fractures that open nearby. The direction is another factor that influences the fracturing of a cell. Another major difference between this section's model and the preliminary version in Section 3.1 is that it considers porosity, which can be defined as the ratio of pore volume (in this case the spaces because of fracturing) to the total volume (both pores and rock volume). Analyzing the neighborhood of the cells (9 cells) to achieve typical porosity values obtainable in Canada (from 15% to 30%) (Natural Resources Canada 2017), we ensured that the number of fractured cells in the neighborhood must not exceed 3.

The model uses the following rules:

- **Fracture rule:** A cell fractures if its port value is 0 (i.e., it is a rock cell), any of its neighbors have the same direction as this cell and the pressure of its neighbor or neighbors exceed its maximum resistance factor. If a cell is fractured (i.e. its state port = 1), the pressure of that cell is computed the average pressure of all the neighborhood cells that are fractured cells. Otherwise, the cell keeps its current value.
- **Well spout rule:** This cell is constantly incrementing its pressure by 10 MPa, generating the pressure required to create the fracture. It also follows the fractured cell rule.
- **Direction rule:** The direction port of each cell constantly holds a random value between 0 and 8 every 100 ms. We chose this range for a random number generator as there are 8 neighbors to each cell – Moore neighborhood and increases the randomness of the direction of each cell, reducing the probability of clustering.

The model presented in Figure 6 shows the results of what we obtained when we considered the porosity of the rock cells. Porosity is another factor, expressed as the percentage of the total rock which is taken up by pore spaces. For example, a sandstone rock may have an 8% porosity. This means that 92% is solid rock and 8% is open space containing oil, gas or water. 8% is about the minimum porosity required. For a fractured formation, we expect porosity values of about 20 to 30 percent (Cipolla et al. 2010).

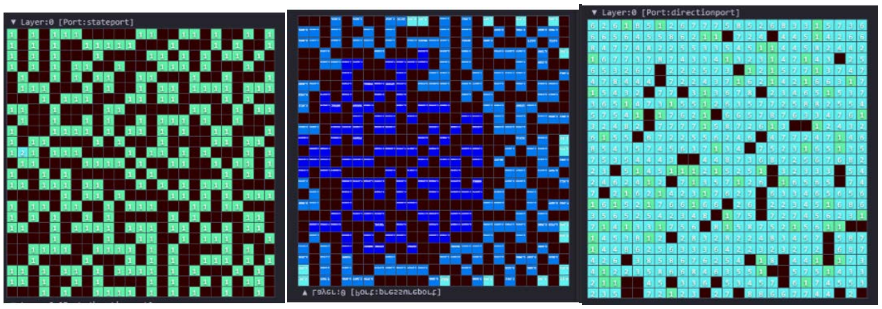

Figure 6: Simulation results (25x25 cell space) with fracture terminating conditions.

When we simulate the model, we obtain the results presented in Figure 7. The plane shows the pressure labelled as Layer 0- [Pressure]. In this layer, blue is a representation of cells which have converted from rocks to fractured cells and those in dark brown represents cells that are still rock cells. The different shades of blue differentiate higher pressure values from lower pressure values (the darker, the higher the pressure is). For all representations and for all cells in the region of cell space, the extreme left cell space shows the values held by the state port, the center cell space shows the values held by the pressure port and the extreme right cell space shows the values held by the direction port.

Figure 7 represents the state, pressure and direction ports of plane 0. This version of the hydraulic fracture model implements fracture stopping conditions. The rules that implement this model interpret that, as the fracture propagates through the rock medium, the pressure decreases by a calculated amount and the fracture will terminate at the point where the pressure of the neighboring cells becomes less than the rock's resistance factor. Figure 8 shows the results for a 100x100 cell space.

The version of the model described in this section captures certain features of the hydraulic fracturing process such as porosity and a larger cell space which shows the fracture patterns better.

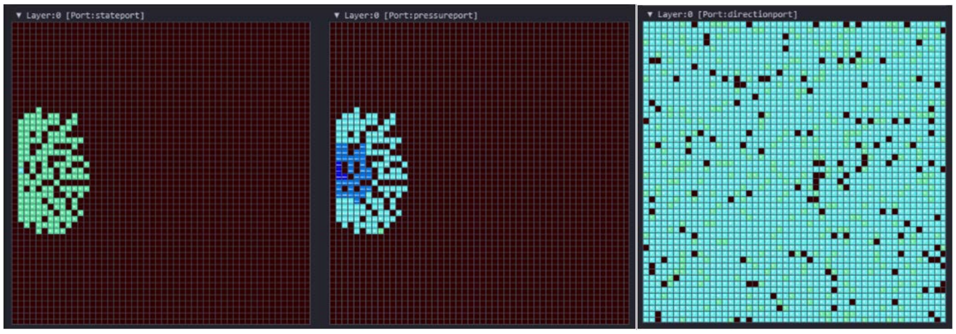

Figure 7: Simulation results (50x50 cell space)

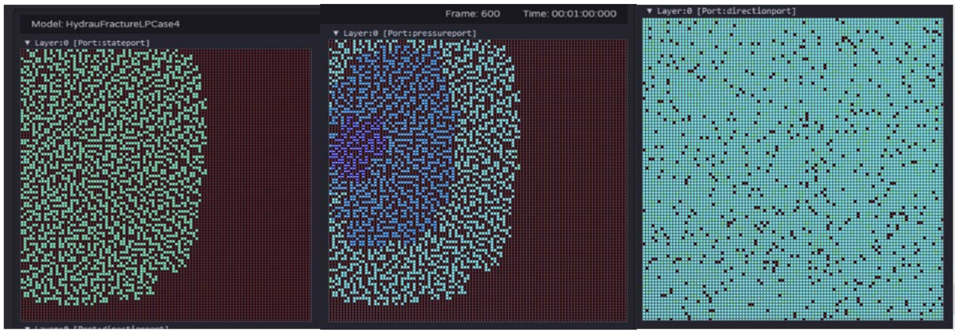

Figure 8: Simulation results (100x100 cell space)

4 CONCLUSION

We presented a Cell-DEVS model of Hydraulic Fracturing processes. The model we presented was implemented in CD++. We showed the results of two implementations of the model Hydraulic Fracturing model. The next steps include defining a direction port can be obtained using real-life data that can be mapped to a statistical distribution. In addition, the use of rectangular meshes has limited applications especially when it comes to fluids and grains of sand and rocks, hence it would be important to consider unstructured meshes for the modelling of fractures.

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